Title: Space Station Air Evacuation Time

Team:504 Problem: A

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Abstract

In this study, we rigorously investigated the influence of pore size on the duration of pressure drop in the context of air leakage through a micro meteor-induced perforation in a space station. Utilizing a well-defined set of assumptions, we formulated a model to quantify the gas overflow velocity and the area of the aperture, enabling us to calculate the time required for the pressure to decrease from 1 atmosphere to 0.3 atmospheres. Through the development of a comprehensive mathematical model, we elucidated the interrelationship between leakage rate, aperture size, and the time necessary for pressure reduction, employing MATLAB for visualization. Our findings indicate that, for an aperture size of 1 cm, the time for pressure to decrease is 7.8 hours. Additionally, we derived mathematical expressions to assess the impact of varying aperture sizes on the pressure reduction dynamics.

Keywords: Leakage Rate, Overflow Velocity, Pore Size, Time.

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1. Introduction

1.1 Backgroug

1.1.1 History

On August 30, 2018, the Ground Mission Control Center identified a minor air leak in the International Space Station. A hole approximately 2 millimeters in diameter was detected in the spacecraft's orbital module and was temporarily sealed with specialized tape. The station's pressure is currently decreasing at a rate of 0.8 millimeters of mercury per hour, posing no immediate threat to crew safety. However, without intervention, the air supply will be exhausted within 18 days. To mitigate risks associated with gas overflow and pressure loss, we aim to investigate the relationship between aperture size and pressure drop to establish the optimal safe escape time.

1.1.2 Leakage Rate

The leakage rate refers to the volume of dry gas, at a specified temperature, that flows through a leakage point per unit time, depending on the pressure difference across the point. For instance, consider a vacuum container sealed by a plug valve, which features a leakage hole with a diameter of 1 cm in its wall. The external environment is at atmospheric pressure, while the interior is under vacuum. When the valve is suddenly opened, all air molecules within the cylindrical space measuring 0.39 inches (1 cm) in diameter and 1082 feet (330 m) in height will rush into the leak at the speed of sound (330 m/s) within one second.

2 Notation

SYMBOLS	DESCRIPTION
$\triangle$**P**	pressure difference
P	gas density
V	gas ejection velocity
Q	leakage rate
S	hole area
v_c	sound velocity
γ	Adiabatic index
R	molar gas constant
M	Molar mass of gas molecule
T	Gas temperature
V	Space Station Volume
L	Internal length of space station
D	Space station diameter

Here the main notations are defined while their specific values will be discussed and given later

3 Model and Formulas

To estimate the time required for the air pressure inside a space station to decrease from 1 atmosphere (atm) to 0.3 atm due to a hole created by a micrometeorite, we must analyze the fluid dynamics associated with the gas flow through the hole. This involves applying Bernoulli's equation and the continuity equation; however, for simplicity, we will utilize an approximate model grounded in the ideal gas law and flow rate principles.

Here is a step-by-step approach to estimate the time:

3.1 Temperature

In a flowing gas, the propagation speed of weak disturbances relative to the airflow is equivalent to the speed of sound. In a non-uniform temperature field, the speed of sound varies at different points, and the speed corresponding to the local temperature is referred to as the "local speed of sound." In conditions of high airflow temperature (such as hypersonic flow) or when influenced by external excitation, the kinetic energy of molecular vibrations increases, leading to a higher degree of molecular dissociation. In this simulation environment, the atmospheric temperature is maintained at a constant 20 °C.

3.2 Identify the dimensions and conditions

The space station is designed as a cylinder with the following dimensions:

Heigh h=50 meters

Diameter D=4 meters

The air inside the space station is at: The air inside the space station has the following conditions:

Temperature T=20C (which is 293 K)

Pressure P_1=1 atmosphere (101.3 kPa)

Additionally, there is a hole with a diameter of D=2**r**=1cm(0.01 meters).

To calculate the volume V of the space station, we use the formula for the volume of a cylinder:

$$V = \pi(\frac{D}{2})^2 h$$

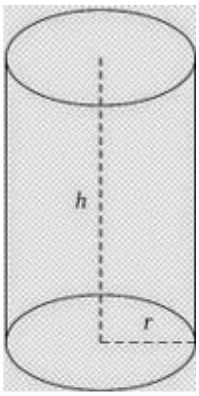

Figure 1

3.3 Torricelli's law

Torricelli's law is a specific application of Bernoulli's principle, derived under the assumption of an ideal fluid (incompressible and inviscid). In real-world scenarios, the actual flow velocity may be slightly lower than the theoretical velocity predicted by the formula.

The formula assumes that the small hole is sufficiently small, allowing the rate of decrease in volume to be neglected relative to the outflow velocity from the hole. In this simulation, the diameter of the hole is 0.1 meters, which is significantly smaller than the overall volume.

$$\mathrm{v} = \sqrt{\frac{2\Delta P}{\rho}}$$

For the entire system comprising the space station and the vacuum environment

$$\mathrm{v} = \sqrt{\frac{2P}{\rho}}$$

In summary, the leakage rate can be expressed as follows:

$$Q = \sqrt{\frac{2P}{\rho}} SP$$

3.4 Determination of Sound Velocity Flow

Based on the literature review, it is known that there exists a sound velocity flow state in a vacuum. Specifically, when the gas ejection speed exceeds the speed of sound, it is accompanied by complex effects that ultimately cause the gas speed to approach the speed of sound.

Therefore, we need to determine whether the ejection speed is close to the speed of sound**.**The air inside the space station is at a temperature of 20 C and a pressure of 1 atmosphere (101.3 kPa). In this environment, the speed of sound is as follows:

$$\mathrm{v}_{\mathrm{c}} = \sqrt{\frac{\gamma RT}{M}}$$

Substituting the data yields v_c=343.2m/s

Due to existence：

$$\rho = \frac{PM}{RT}$$

Combining the above equation, it can be concluded that：

$$\mathrm{v} = \sqrt{\frac{2P}{\rho}} = \sqrt{\frac{2RT}{M}}$$

Obtain a spray velocity of v=409.9m/s＞343.2m/s

The final result is that the ejection velocity is always the speed of sound, that is:

$$\mathrm{v} = \mathrm{v}_{\mathrm{c}}$$

$$Q = \mathrm{v}_{\mathrm{c}} SP$$

3.5 The relationship between change in pressure and time

We postulate that the gas within the space station is an ideal gas and the temperature remains constantly at 20 degrees Celsius. Then the gas within the space station always complies with the ideal gas equation of state, namely xxx. Due to the invariable temperature, xx is proportional to n. During a gas leak on the space station, the pressure of the gas is constantly changing, so calculus is needed to describe the process.

Let's say the change in pressure over a very short period of time is dP

$$V\left(\left(P-dP\right)-P\right)=Qdt$$
$$-VdP=Qd\mathrm{t}$$

plug $Q=\mathrm{v}_{\mathrm{c}}SP$ into the formula

$$-VdP=v_cSPdt$$
$$\frac{1}{P}dP=-\frac{v_cS}{V}dt$$

Integration

$$ln\left(P\right)=-\frac{v_cS}{V}t+ln\left(P_0\right)$$
$$t=\frac{V}{v_cS}\ln\left(\frac{P_0}{P}\right)$$

Substituting the data, we obtain:

$$t=28064.63s$$

In other words, the gas in the space station will drop to 0.3 atmospheres in approximately 7.8 hours.

4 Strength&Weakness

4.1Strength

1. We reached this conclusion through rigorous mathematical proof and derivation.
2. We also compared our findings with the history of spacecraft accidents..

4.2Weakness

1. We utilized a single model for derivation instead of integrating multiple models, which may have resulted in the oversight of potential influencing factors.
2. Assuming gas leakage speed equals the speed of sound is an oversimplification.

5 Results

5.1 Leakage time

We find out how long it takes for the air to escape under the conditions given in the problem. 7.8 hours

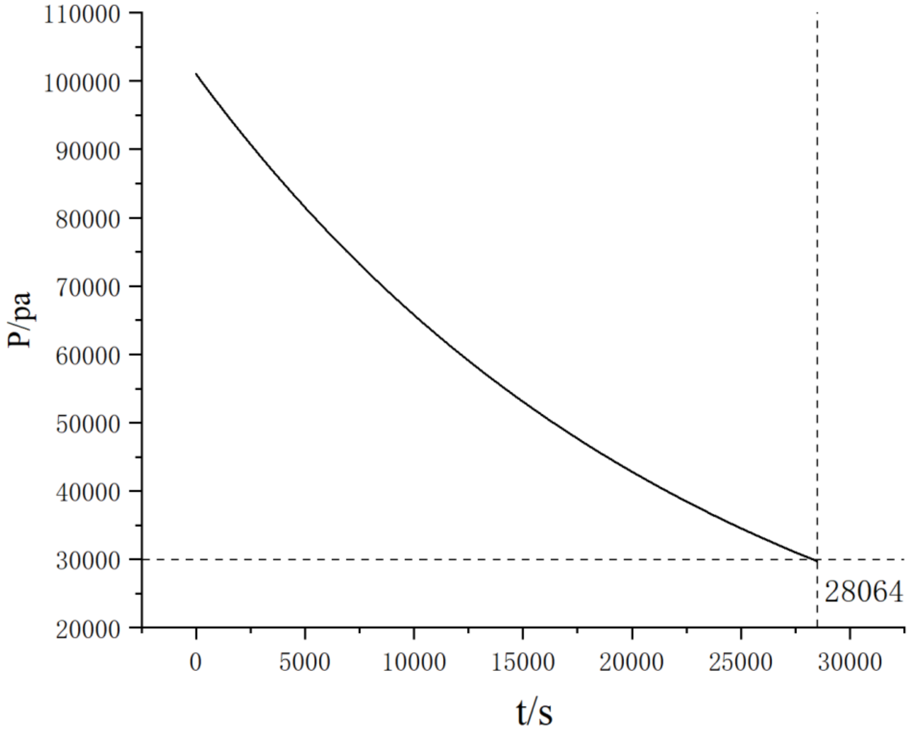

Figure 2 Graph of the variation of air pressure over time in the space station.

5.2 The relationship between leakage time and hole diameter

$$t = \frac{V}{v_c S} \ln\left(\frac{P_0}{P}\right)$$

Under constant conditions, the time required for leakage is inversely proportional to the controlled area. In other words, the leakage time is inversely proportional to the square of the aperture size.

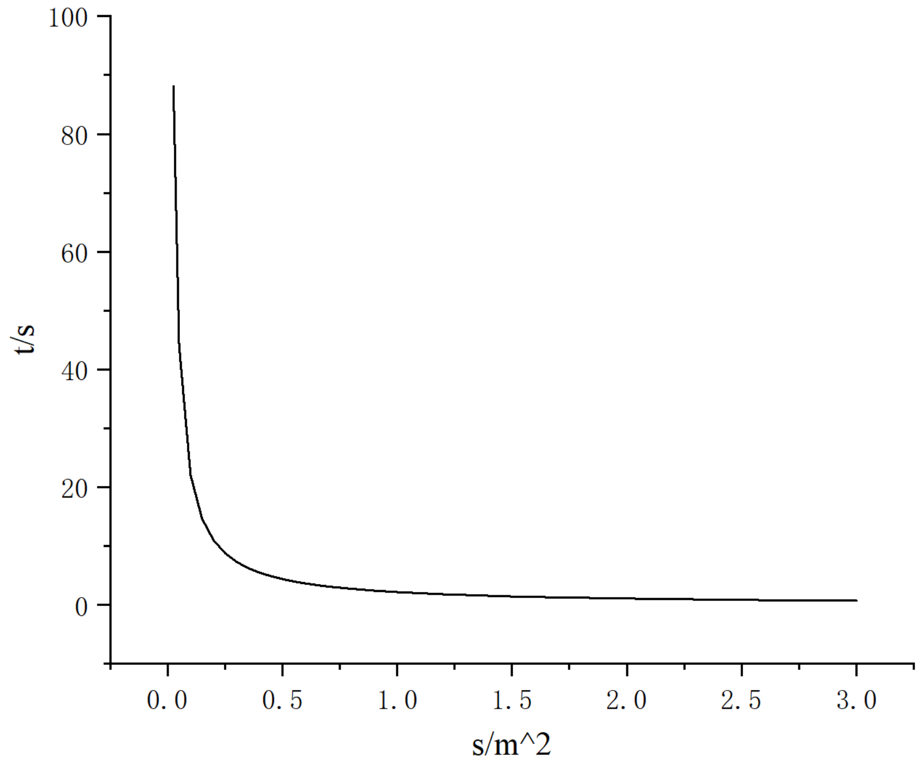

Figure 3Relationship between leak time and hole area.

6 Conclusion

This paper analyzes and discusses the issue of leak time in the space station, considering factors such as the state of gases within the station, the flow dynamics of the ejected gas, and the pressure difference between the inside and outside. We assume that the ejected gas behaves as a sonic flow and introduce the concept of leakage rate, calculating the expression for the leakage rate under sonic flow conditions. By establishing a differential equation relating pressure P and time t , we derive the function of pressure P with respect to time t . Ultimately, we find that the time required for the space station to decrease from 1 atmosphere to 0.3 atmospheres is approximately 7.8 hours, and we conclude that the leakage time is inversely proportional to the square of the aperture size.

7 References

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